



# Bistatic scattering from sea surfaces at non-near-specular direction via the Polarimetric Two-Scale Model for maritime surveillance with GNSS-R

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# Outline

- ✓ Introduction and motivation
- ✓ Theory: PTSM
- ✓ Results
- ✓ Conclusions

# *Introduction and motivation*

Recent increasing interest in the application of GNSS-R to the field of **maritime surveillance** and **ship detection**.

**Examples:**

- GNSS-R signals for maritime surveillance from **fixed** receivers **located on the coast**<sup>1</sup>.
- detection of **large structures** over the ocean using U.K. TechDemoSat-1 delay-Doppler maps (DDMs)<sup>2</sup>

**But**

current spaceborne GNSS-R systems are **not** suited for **ship detection** applications.

<sup>1</sup>H. Ma, M. Antoniou, A. G. Stove, J. Winkel and M. Cherniakov, “Maritime Moving Target Localization Using Passive GNSS-Based Multistatic Radar,” in IEEE Transactions on Geoscience and Remote Sensing, vol. 56, no. 8, pp. 4808-4819, Aug. 2018.

<sup>2</sup>A. Di Simone, H. Park, D. Riccio, and A. Camps, “Sea target detection using spaceborne GNSS-R delay-Doppler maps: Theory and experimental proof of concept using TDS-1 data,” *IEEE J. Sel. Topics Appl. Earth Observ.*, **10**, 9, pp. 4237-4255 (2017).

# *Introduction and motivation*

Current spaceborne GNSS-R systems have been conceived for sea state monitoring:

Sufficient SNR  receiver designed to acquire from the **specular** or **near-specular** area on the sea

For **ship detection** this is the **worst** situation: **low signal-to-clutter**, **low resolution**

**non-near-specular** acquisition geometry would be desirable!

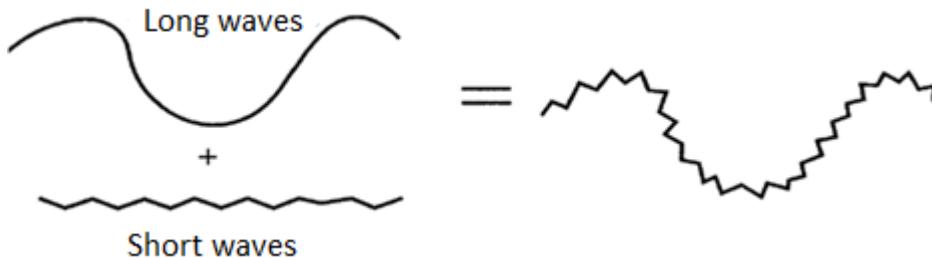
To be assessed via **simulation**: **scattering models** for ship and sea surface

Scattering from the sea at **non-near-specular** direction: Two-Scale Model

**Closed form** available in backscattering. Here: **extension to bistatic scattering**

# Theory: PTSM

## Two-Scale Model (TSM)



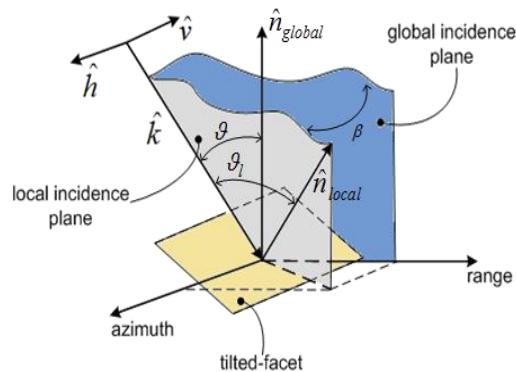
Total NRCS = **large scale** roughness NRCS (computed via GO) + **small scale** roughness NRCS (computed via SPM)

The GO term dominates at near-specular directions, the SPM term dominates at non-near-specular directions

Range of validity: union of GO and SPM ones.

**no cross-pol and de-pol unless SPM term is averaged over random slopes of tilted mean plane**

Average over slopes → **numerical integration!**



J. W. Wright, "A New Model for Sea Clutter", *IEEE Trans. Antennas Propagat.*, vol. 16, pp. 217-223, 1968.

G. R. Valenzuela, "Scattering of Electromagnetic Waves from a Tilted Slightly Rough Surface", *Radio Sci.*, vol. 3, pp. 1057-1066, 1968.

# Theory: PTSM

- Recently<sup>1</sup> we showed how to analytically compute **in closed form** the average integral, with a moderate slope approximation, **in the backscattering case**, so introducing the **Polarimetric Two-Scale Model (PTSM)**
- PTSM allows accounting for **cross-** and **de-polarisation** effects actually present in measured data even when **surface scattering** is the **only** present mechanism.
- Based on it, we devised a soil moisture retrieval scheme for **bare soils**, making use of the SAR measured **copol** and **crosspol ratios**<sup>1</sup> (or **copol ratio** and **correlation coefficient**<sup>2</sup>)
- **Generalization to the bistatic case (and to sea surface) is needed.**

<sup>1</sup> A. Iodice, A. Natale, D. Riccio, “Retrieval of Soil Surface Parameters via a Polarimetric Two-Scale Model”, *IEEE Trans. Geosci. Remote Sens.* vol. 49, no. 7, pp. 2531-2547, July 2011.

<sup>2</sup> A. Iodice, A. Natale, D. Riccio, “Polarimetric Two-Scale Model for Soil Moisture Retrieval via Dual-Pol HH-VV SAR Data”, *IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens.* vol. 11, no. 3, pp. 1163-1171, June 2013.

# Theory: PTSM

## Surface description

1) Small-scale roughness PSD: directional Elfouhaily spectrum  $W_{2D}(\kappa, \varphi) = W(\kappa)\Phi(\kappa, \varphi)$

$$W(\kappa) = \frac{\pi\alpha_m c_m}{c \kappa^4} \exp\left[-\frac{1}{4}\left(\frac{\kappa}{\kappa_m} - 1\right)^2\right] \cong \frac{S_0}{\kappa^{3.5}}$$

$$\Phi(\kappa, \varphi) = 1 + \Delta(\kappa) \cos[2(\varphi_w - \varphi)]$$

2) Large-scale roughness slopes,  $s_x$  and  $s_y$ : correlated zero-mean jointly Gaussian variables

$$s_x, s_y \sim N(0; \sigma_x^2, \sigma_y^2, \rho)$$

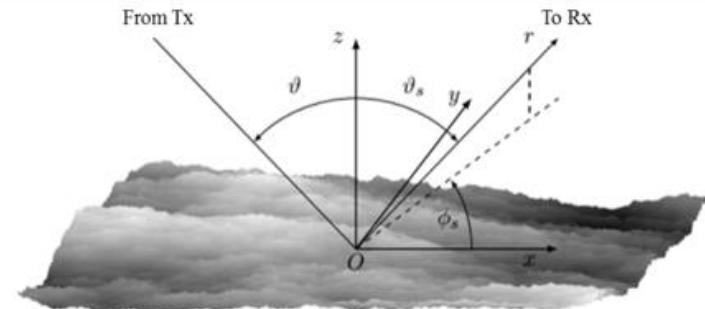
$$\sigma_{up}^2 = 0.45 [0.00316 \cdot 6 \ln(u_{10})]$$

$$\sigma_{cross}^2 = 0.45 [0.003 + 0.00192 \cdot 6 \ln(u_{10})]$$

$$\sigma_x^2 = \sigma_{up}^2 \cos^2 \varphi_w + \sigma_{cross}^2 \sin^2 \varphi_w$$

$$\sigma_y^2 = \sigma_{cross}^2 \cos^2 \varphi_w + \sigma_{up}^2 \sin^2 \varphi_w$$

$$\rho = \frac{1}{2} \sin 2\varphi_w \frac{\sigma_{cross}^2 - \sigma_{up}^2}{\sigma_r \sigma_a}$$

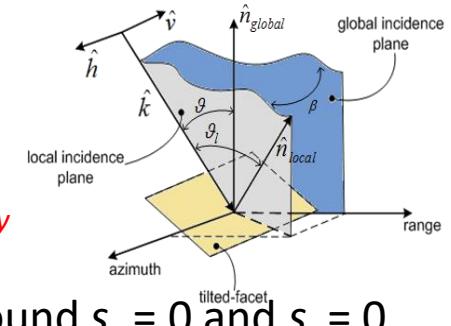


Isotropic approximation:

$$\Phi(\kappa, \varphi) \cong 1 \quad \sigma_x = \sigma_y = \frac{\sigma_{up} + \sigma_{cross}}{2}, \quad \rho = 0$$

# Theory: PTSM

- 1) Compute tilted surface's polarimetric covariance matrix via SPM in terms of the local incidence  $\vartheta_{li}$  and scattering  $\vartheta_{ls}$ ,  $\varphi_{ls}$  angles, and of rotation angles  $\beta_i$  and  $\beta_s$  of incidence and scattering planes
- 2) Express  $\vartheta_{li}$ ,  $\vartheta_{ls}$ ,  $\varphi_{ls}$ ,  $\beta_i$  and  $\beta_s$  in terms of global incidence  $\vartheta_i$  and scattering  $\vartheta_s$ ,  $\varphi_s$  angles and of local surface slopes  $s_x$  and  $s_y$
- 3) Second order expansion of tilted surface's covariance matrix around  $s_x = 0$  and  $s_y = 0$
- 4) Averaging tilted surface's NRCS and other entries of the covariance matrix over  $s_x$  and  $s_y$  by using:  $\langle s_x \rangle = \langle s_y \rangle = 0$ ,  $\langle s_x^2 \rangle = \sigma_x^2$ ,  $\langle s_y^2 \rangle = \sigma_y^2$ , and  $\langle s_x s_y \rangle = \rho \sigma_x \sigma_y$



Expressions for  $\vartheta_{li}$  and  $\beta_i$  are already available, while those for  $\vartheta_{ls}$ ,  $\varphi_{ls}$  and  $\beta_s$  are an original contribution of this work

# Theory: PTSM

$$\cos \vartheta_{li} = \frac{\cos \vartheta_i + s_x \sin \vartheta_i}{\sqrt{1 + s_x^2 + s_y^2}} \quad \tan \beta_i = \frac{s_y}{-s_x \cos \vartheta_i + \sin \vartheta_i}$$

$$\cos \vartheta_{ls} = \frac{-s_y \sin \vartheta_s \sin \varphi_s - s_x \sin \vartheta_s \cos \varphi_s + \cos \vartheta_s}{\sqrt{1 + s_x^2 + s_y^2}}$$

$$\cos \varphi_{ls} =$$

$$\frac{(\sin \vartheta_i - s_x \cos \vartheta_i)(\sin \vartheta_s \cos \varphi_s + s_x \cos \vartheta_s) + s_y \cos \vartheta_i(-s_y \cos \vartheta_s - \sin \vartheta_s \sin \varphi_s) + s_y \sin \vartheta_i(-s_x \sin \vartheta_s \sin \varphi_s + s_y \sin \vartheta_s \cos \varphi_s)}{\sqrt{(\sin \vartheta_i - s_x \cos \vartheta_i)^2 + s_y^2} \cdot \sqrt{(\sin \vartheta_s \cos \varphi_s + s_x \cos \vartheta_s)^2 + (-s_y \cos \vartheta_s - \sin \vartheta_s \sin \varphi_s)^2 + (-s_x \sin \vartheta_s \sin \varphi_s + s_y \sin \vartheta_s \cos \varphi_s)^2}}$$

$$\tan \beta_s = \frac{s_y \cos \varphi_s - s_x \sin \varphi_s}{s_y \cos \vartheta_s \sin \varphi_s + s_x \cos \vartheta_s \cos \varphi_s + \sin \vartheta_s}$$

# Theory: PTSM

## Linear polarization basis

$$\left\{ \begin{array}{l} \langle R_{hhhh} \rangle|_{sx,sy} = C_{0,0}^{hhhh} + \sigma^2 \left[ C_{0,2}^{hhhh} + C_{2,0}^{hhhh} + \frac{C_{0,0}^{vhvh} - C_{0,0}^{hhhh}}{\sin^2 \vartheta_s} + \frac{C_{0,0}^{hvvh} - C_{0,0}^{hhhh}}{\sin^2 \vartheta_i} + \frac{2 \cos \varphi_s (Re\{C_{0,0}^{hhvv}\} + Re\{C_{0,0}^{hvvh}\})}{\sin \vartheta_s \sin \vartheta_i} \right] \\ \langle R_{vvvv} \rangle|_{sx,sy} = C_{0,0}^{vvvv} + \sigma^2 \left[ C_{0,2}^{vvvv} + C_{2,0}^{vvvv} + \frac{C_{0,0}^{hvvh} - C_{0,0}^{vvvv}}{\sin^2 \vartheta_s} + \frac{C_{0,0}^{vhvh} - C_{0,0}^{vvvv}}{\sin^2 \vartheta_i} + \frac{2 \cos \varphi_s (Re\{C_{0,0}^{hhvv}\} + Re\{C_{0,0}^{hvvh}\})}{\sin \vartheta_i \sin \vartheta_s} \right] \\ \langle R_{hvvh} \rangle|_{sx,sy} = C_{0,0}^{hvvh} + \sigma^2 \left[ C_{0,2}^{hvvh} + C_{2,0}^{hvvh} + \frac{C_{0,0}^{hhhh} - C_{0,0}^{hvvh}}{\sin^2 \vartheta_i} + \frac{C_{0,0}^{vvvv} - C_{0,0}^{hvvh}}{\sin^2 \vartheta_s} - \frac{2 \cos \varphi_s (Re\{C_{0,0}^{hhvv}\} + Re\{C_{0,0}^{hvvh}\})}{\sin \vartheta_i \sin \vartheta_s} \right] \\ \langle R_{vhvh} \rangle|_{sx,sy} = C_{0,0}^{vhvh} + \sigma^2 \left[ C_{0,2}^{vhvh} + C_{2,0}^{vhvh} + \frac{C_{0,0}^{vvvv} - C_{0,0}^{vhvh}}{\sin^2 \vartheta_i} + \frac{C_{0,0}^{hhhh} - C_{0,0}^{vhvh}}{\sin^2 \vartheta_s} - \frac{2 \cos \varphi_s (Re\{C_{0,0}^{hhvv}\} + Re\{C_{0,0}^{hvvh}\})}{\sin \vartheta_i \sin \vartheta_s} \right] \\ \langle R_{hhvv} \rangle|_{sx,sy} = C_{0,0}^{hhvv} + \sigma^2 \left[ C_{0,2}^{hhvv} + C_{2,0}^{hhvv} - \frac{C_{0,0}^{hvvh} + C_{0,0}^{hhvv}}{\sin^2 \vartheta_i} - \frac{C_{0,0}^{vhvh} + C_{0,0}^{hhvv}}{\sin^2 \vartheta_s} + \frac{\cos \varphi_s (C_{0,0}^{hhhh} - C_{0,0}^{hvvh} - C_{0,0}^{vhvh} + C_{0,0}^{vvvv})}{\sin \vartheta_i \sin \vartheta_s} \right] \\ \langle R_{hvvh} \rangle|_{sx,sy} = C_{0,0}^{hvvh} + \sigma^2 \left[ C_{0,2}^{hvvh} + C_{2,0}^{hvvh} + \frac{C_{0,0}^{hhvv} - C_{0,0}^{hvvh}}{\sin^2 \vartheta_i} + \frac{C_{0,0}^{vhvh} - C_{0,0}^{hvvh}}{\sin^2 \vartheta_s} + \frac{\cos \varphi_s (C_{0,0}^{hhhh} - C_{0,0}^{vhvh} - C_{0,0}^{hvvh} + C_{0,0}^{vvvv})}{\sin \vartheta_i \sin \vartheta_s} \right] \\ \langle R_{hhh} \rangle|_{sx,sy} = C_{0,0}^{hhh} + \sigma^2 \left[ C_{0,2}^{hhh} + C_{2,0}^{hhh} + \frac{-C_{0,0}^{hvvh} - C_{0,0}^{hhh}}{\sin^2 \vartheta_i} + \frac{C_{0,0}^{vhvh} - C_{0,0}^{hhh}}{\sin^2 \vartheta_s} + \frac{\cos \varphi_s (C_{0,0}^{hvvh} - C_{0,0}^{vhvh} - C_{0,0}^{vhvh} + C_{0,0}^{vvvh})}{\sin \vartheta_i \sin \vartheta_s} \right] \\ \langle R_{vvvh} \rangle|_{sx,sy} = C_{0,0}^{vvvh} + \sigma^2 \left[ C_{0,2}^{vvvh} + C_{2,0}^{vvvh} + \frac{C_{0,0}^{vhvh} - C_{0,0}^{vvvh}}{\sin^2 \vartheta_i} + \frac{-C_{0,0}^{hvvh} - C_{0,0}^{vvvh}}{\sin^2 \vartheta_s} + \frac{\cos \varphi_s (C_{0,0}^{hhh} - C_{0,0}^{vhvh} - C_{0,0}^{vvvh} + C_{0,0}^{hvvh})}{\sin \vartheta_i \sin \vartheta_s} \right] \end{array} \right.$$

$$\left\{ \begin{array}{l} F_{hh} = \frac{(\varepsilon_r - 1) \cos \varphi_s}{(\cos \varphi_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s})(\cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i})} \\ F_{hv} = \frac{\sin \varphi_s [(\varepsilon_r - 1)(\sqrt{\varepsilon_r - \sin^2 \vartheta_s})]}{(\sqrt{\varepsilon_r - \sin^2 \vartheta_s} + \varepsilon_r \cos \vartheta_s)(\cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i})} \\ F_{vh} = \frac{\sin \varphi_s [(\varepsilon_r - 1)(\sqrt{\varepsilon_r - \sin^2 \vartheta_i})]}{(\sqrt{\varepsilon_r - \sin^2 \vartheta_i} + \varepsilon_r \cos \vartheta_i)(\cos \vartheta_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s})} \\ F_{vv} = \frac{(\varepsilon_r - 1)[\sqrt{\varepsilon_r - \sin^2 \vartheta_i} \sqrt{\varepsilon_r - \sin^2 \vartheta_s} \cos \varphi_s - \varepsilon_r \sin \vartheta_i \sin \vartheta_s]}{(\varepsilon_r \cos \vartheta_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s})(\varepsilon_r \cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i})} \end{array} \right.$$

$$C_{k,n-k}^{pqrs} = \frac{1}{n!} \binom{n}{k} \left. \frac{\partial^2 (W k^4 \cos^2 \vartheta_{li} \cos^2 \vartheta_{ls} F_{pq} F_{rs}^*)}{\partial a^k \partial b^{n-k}} \right|_{sx=sy=0}$$

# Theory: PTSM

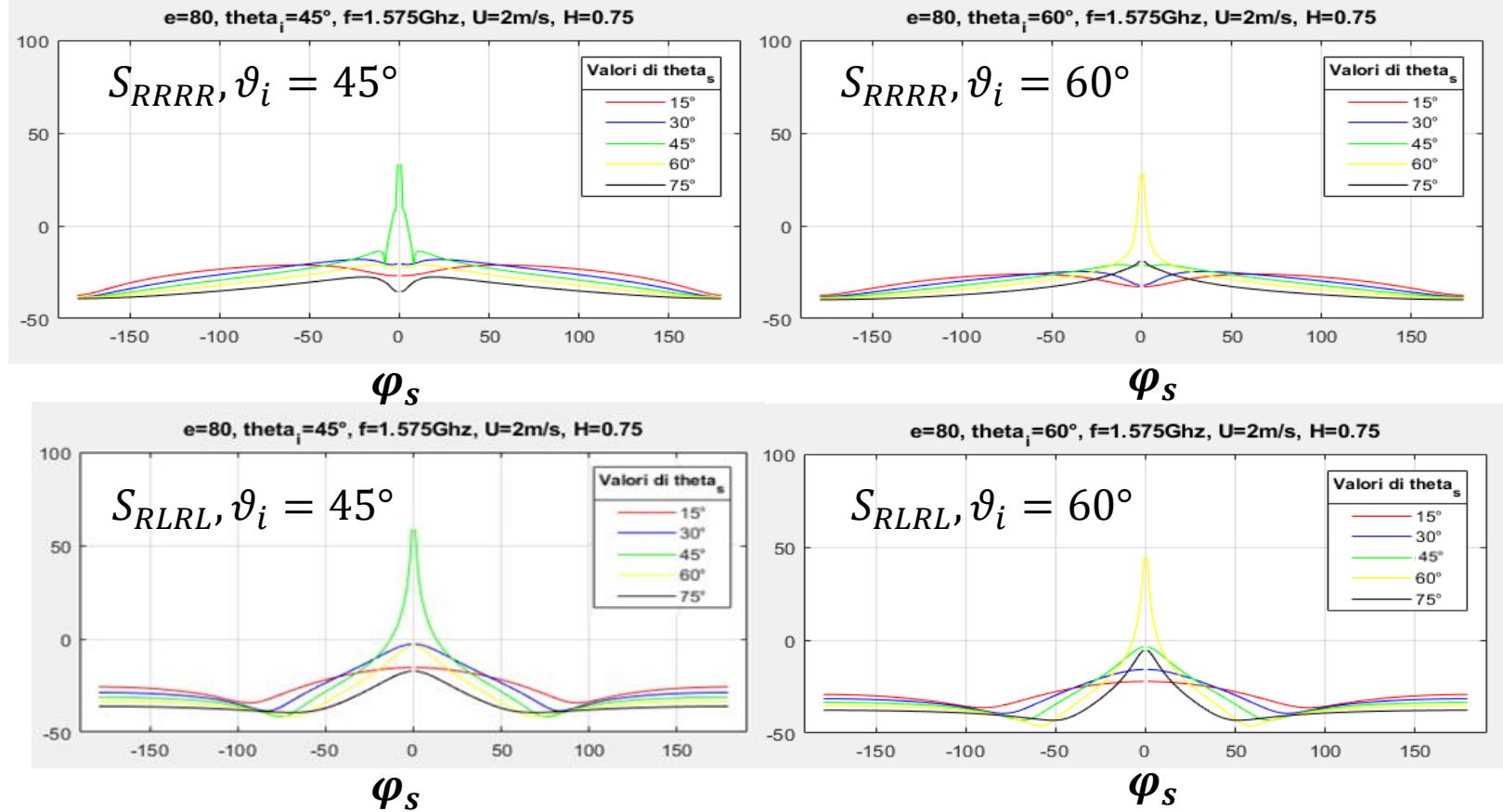
## Circular polarization basis

$$\begin{cases} F_{RR} = (F_{hh} - F_{vv} + iF_{hv} + iF_{vh})/2 \\ F_{LL} = (F_{vv} - F_{hh} + iF_{hv} + iF_{vh})/2 \\ F_{RL} = (iF_{hh} + iF_{vv} + F_{hv} - F_{vh})/2 \\ F_{LR} = (iF_{hh} + iF_{vv} - F_{hv} + F_{vh})/2 \end{cases}$$

$$\begin{cases} |\chi_{RR}|^2 = \frac{1}{4}(|F_{hh}|^2 + |F_{vv}|^2 + |F_{hv}|^2 + |F_{vh}|^2 - 2Re\{F_{hh}F_{vv}^*\} + 2Re\{F_{hv}F_{vh}^*\} + 2Im\{F_{hh}F_{hv}^*\} + 2Im\{F_{hh}F_{vh}^*\} - 2Im\{F_{vv}F_{hv}^*\} - 2Im\{F_{vv}F_{vh}^*\}) \\ |\chi_{RL}|^2 = \frac{1}{4}(|F_{hh}|^2 + |F_{vv}|^2 + |F_{hv}|^2 + |F_{vh}|^2 + 2Re\{F_{hh}F_{vv}^*\} - 2Re\{F_{hv}F_{vh}^*\} - 2Im\{F_{hh}F_{hv}^*\} + 2Im\{F_{hh}F_{vh}^*\} - 2Im\{F_{vv}F_{hv}^*\} + 2Im\{F_{vv}F_{vh}^*\}) \\ \chi_{RR}\chi_{LL}^* = \frac{1}{4}(-|F_{hh}|^2 - |F_{vv}|^2 + |F_{hv}|^2 + |F_{vh}|^2 + 2Re\{F_{hh}F_{vv}^*\} + 2Re\{F_{hv}F_{vh}^*\} - 2iRe\{F_{hh}F_{hv}^*\} - 2iRe\{F_{hh}F_{vh}^*\} + 2iRe\{F_{vv}F_{hv}^*\} + 2iRe\{F_{vv}F_{vh}^*\})e^{-i2(\beta_i + \beta_s)} \\ \chi_{RL}\chi_{LR}^* = \frac{1}{4}(|F_{hh}|^2 + |F_{vv}|^2 - |F_{hv}|^2 - |F_{vh}|^2 + 2Re\{F_{hh}F_{vv}^*\} + 2Re\{F_{hv}F_{vh}^*\} - 2iRe\{F_{hh}F_{hv}^*\} + 2iRe\{F_{hh}F_{vh}^*\} - 2iRe\{F_{vv}F_{hv}^*\} + 2iRe\{F_{vv}F_{vh}^*\})e^{i2(\beta_i - \beta_s)} \\ \chi_{RR}\chi_{RL}^* = \frac{1}{4}(-i|F_{hh}|^2 + i|F_{vv}|^2 + i|F_{hv}|^2 - i|F_{vh}|^2 + 2Im\{F_{hh}F_{vv}^*\} + 2Im\{F_{hv}F_{vh}^*\} + 2Re\{F_{hh}F_{hv}^*\} - 2iIm\{F_{hh}F_{vh}^*\} - 2iIm\{F_{vv}F_{hv}^*\} + 2Re\{F_{vv}F_{vh}^*\})e^{-i2\beta_i} \\ \chi_{RR}\chi_{LR}^* = \frac{1}{4}(-i|F_{hh}|^2 + i|F_{vv}|^2 - i|F_{hv}|^2 + i|F_{vh}|^2 + 2Im\{F_{hh}F_{vv}^*\} - 2Im\{F_{hv}F_{vh}^*\} - 2iIm\{F_{hh}F_{hv}^*\} + 2Re\{F_{hh}F_{vh}^*\} + 2Re\{F_{vv}F_{hv}^*\} - 2iIm\{F_{vv}F_{vh}^*\})e^{-i2\beta_s} \\ \chi_{RL}\chi_{RR}^* = \frac{1}{4}(i|F_{hh}|^2 - i|F_{vv}|^2 - i|F_{hv}|^2 + i|F_{vh}|^2 + 2Im\{F_{hh}F_{vv}^*\} + 2Im\{F_{hv}F_{vh}^*\} + 2Re\{F_{hh}F_{hv}^*\} + 2iIm\{F_{hh}F_{vh}^*\} + 2iIm\{F_{vv}F_{hv}^*\} + 2Re\{F_{vv}F_{vh}^*\})e^{i2\beta_i} \\ \chi_{RL}\chi_{LL}^* = \frac{1}{4}(-i|F_{hh}|^2 + i|F_{vv}|^2 - i|F_{hv}|^2 + i|F_{vh}|^2 - 2Im\{F_{hh}F_{vv}^*\} + 2Im\{F_{hv}F_{vh}^*\} + 2iIm\{F_{hh}F_{hv}^*\} - 2Re\{F_{hh}F_{vh}^*\} - 2Re\{F_{vv}F_{hv}^*\} + 2iIm\{F_{vv}F_{vh}^*\})e^{-i2\beta_s} \end{cases}$$

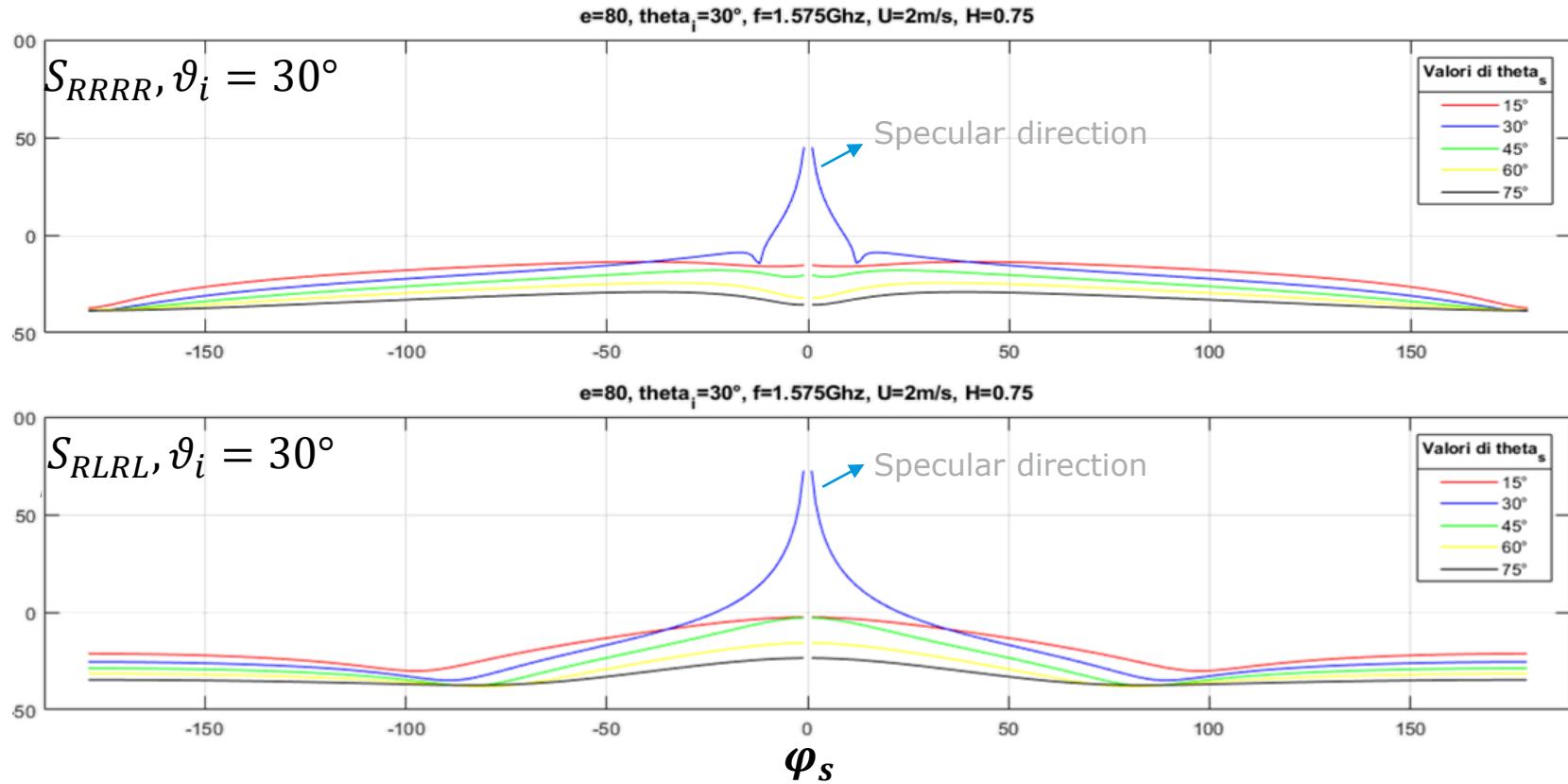
# Results

## Bistatic NRCS vs $\varphi_s$ (isotropic sea)



# Results

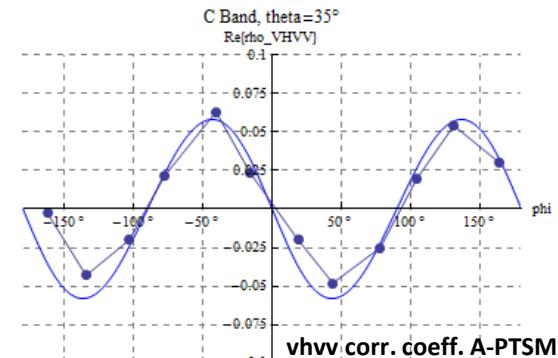
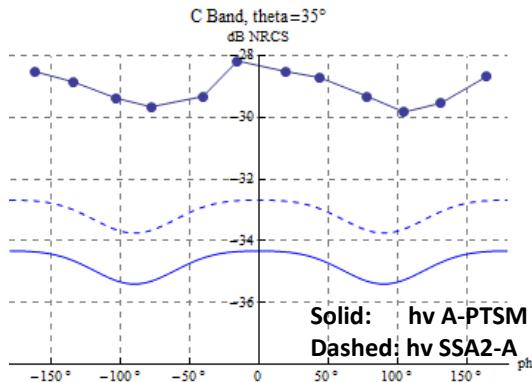
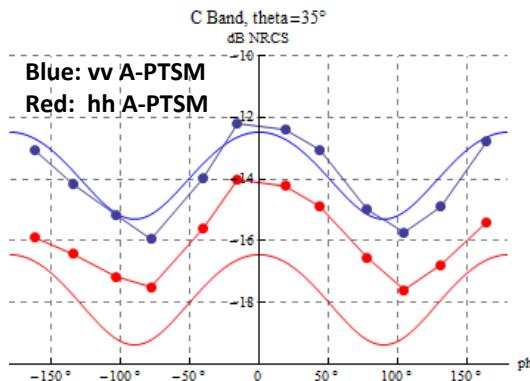
## Bistatic NRCS vs $\varphi_s$ (isotropic sea)



# Results

Monostatic NRCS vs  $\varphi_w$  (anisotropic sea) C band  $u_{10} = 10 \text{ m/s}$  theta=35°

Connected dots: experimental data



To be compared to Fig. 5 of Ref. SSA2, see below

**SSA2:** A. G. Voronovich and V. U. Zavorotny, “Full-polarization modeling of monostatic and bistatic radar scattering from a rough sea surface”, *IEEE Trans. Antennas Propagat.*, vol. 62, no. 3, pp. 1362–1371, March 2014.

**SSA2-A:** C. A. Guérin and J. T. Johnson, “A simplified formulation for rough surface cross-polarized backscattering under the second-order small-slope approximation”, *IEEE Trans. Geosc. Remote Sens.*, vol. 53, no. 11, pp. 6308–6314, Nov 2015.

# *Conclusions*

- Closed-form PTSM extended to the **bistatic case (B-PTSM)** and applied to scattering from the **sea surface**
- **All elements** of the polarimetric covariance matrix analytically expressed in **closed form**, both in the **linear** and in the **circular** polarization bases
- **In backscattering:** Very good agreement with **numerically** evaluated TSM, Reasonable agreement with **SSA2 and experimental data**, except for HV NRCS (as expected)
- **To be done:** validation vs real scattering data for the fully bistatic case, implementation in a simulation tool



Thank you for your  
attention